# PROBABILISTIC PREDICTION OF FLOOR RESPONSE SPECTRA

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#### SYNOPSIS

A simplified probabilistic method of seismic analysis is reviewed and its application to floor response spectra generation is demonstrated through an example of a seismic analysis of a soil-founded nuclear power plant structure. The earthquake input is represented as a limited duration stationary random process with a power spectral density function derived from the USNRC Design Response Spectrum. In addition, a Kanai-Tajimi spectral density shape function is studied as a representation of earthquake input. The results of the random vibration approach are compared with those of the time history method. It is seen that the probabilistic method provides adequate estimate of floor response spectra for design purposes and requires less computational effort than time history methods, thus enabling one to economically perform varicus parametric studies and reduce overall analysis costs.

#### RESUME

On résume une méthode simple d'analyse sismique. Pour fin de comparaison on simule un tremblement de terre comme processus aléatoire ayant une ampleur variable selon une fonction de forme. On démontre que les résultats du comportement d'une centrale nucléaire selon les deux méthodes sont de mêmes grandeurs.

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### INTRODUCTION

Major critical structures of power plants, communication centers, and hospitals provide support for equipment and systems whose continued function after an earthquake is often essential to the public welfare and safety in providing lifeline services for near and long term recovery periods. For earthquake design and qualification of safety-related equipment and systems, seismic input is often specified in terms of floor response spectra. Such floor response spectra are used in conjunction with either a dynamic modal analysis or an equivalent static method to qualify operability or structural integrity of designs.

Building floor response spectra have been typically generated by both semi-empirical (1) and deterministic (time history) methods. Semi-empirical methods can produce satisfactory results for certain limited situations, but their widespread use for all applications introduces uncertainties that may not be acceptable where a high degree of reliability and safety must be assured. Time history methods have been used extensively in the seismic design of nuclear power plants where it has become standard practice to specify the building seismic input in terms of an "artificial" earthquake record whose spectra must envelope a specified plant design ground response spectra. The computational effort and cost involved in the development of this record can be extensive and the time history analysis of a structure to develop floor time histories and floor response spectra increases this cost significantly. Since earthquake records exhibit characteristics of a random process, it is also possible to determine building seismic response and floor response spectra by means of a probabilistic approach. Although seemingly an attractive alternative to other approaches, its practical use in seismic response applications has, in the past, been somewhat limited because of the complexities associated with the treatment of the nonstationarity of the input and response processes as well as the treatment of the distribution of the maximum response. Recently, however, studies by various investigators (2,3,4) have developed methodologies for dealing with this nonstationarity and for providing a rational basis for adopting a more simplified approach based on time invariant assumptions, thus making the method more attractive for practical applications.

It is the purpose of this paper to review the application of probabilistic methods to seismic analysis and to use these methods to develop floor response spectra for a soil-founded concrete structure and compare them with floor response spectra developed from present state-of-the-art time history methods.

#### BACKGROUND

## Input - Response Relation

A block diagram of the input - response process associated with a structure-equipment system in terms of a time history approach is shown in Figure 1. For the seismic analysis of light equipment within a structure, it is common to treat the structure and the equipment separately. The structural response at a particular point may be viewed as the input excitation to the equipment mounted at that point. The building quipment system is excited by a ground motion,  $\ddot{u}(t)$ , of specified duration, s, which can be represented by a weakly stationary, zero mean Gaussian process having a power spectral density (PSD) function  $G_{ii}(\omega)$ . A formal definition of a power spectral density function can be found in Reference 5, but simply stated it can be viewed as the distribution of the power of the ground motion with respect to its frequency content. Having defined the earthquake input in terms of a PSD function, the input-response process for a random vibration approach can be represented as shown in Figure 2. This section will discuss the process in Figure 2 and present a method for its solution.

## Mean Square Acceleration Response

The dynamic behavior of a single degree-of-freedom oscillator z(t), relative to its point of support, can be described by the following second order differential equation:

$$\dot{z}(t) + 2 S_e \omega_e \dot{z}(t) + \omega_e^2 z(t) = - \ddot{x}(t)$$
(1)

where  $\omega_e$  is the natural frequency of the oscillator,  $\varsigma_e$  is the oscillator damping and  $\ddot{x}(t)$  is the acceleration response of the structure at the point of oscillator support. If the primary structure is not too lightly damped, the time variation of the frequency content of the response  $\ddot{x}(t)$  toward the end of the input ground motion is often not significant and its energy content can be conservatively represented by a stationary power spectral density function as follows:

$$G_{ii}(\omega) = G_{ii}(\omega)/H(\omega)/^{2}$$
(2)

where  $|H(\omega)|^2$  is the squared amplitude of the transfer function of the primary system.

Due to the characteristic low damping of building-mounted equipment and systems, the response z(t) in Equation 1 may never achieve a steady state response. It is therefore necessary to define its response by means of a time dependent spectral density function,  $G_{z}(\omega, t)$ . The time dependent power spectral density function,  $G_{z}(\omega, t)$ , of a single degree-offreedom oscillator can be expressed as(6):

$$G_{z}(\omega,t) = G_{z}(\omega) / H_{e}(\omega,t)/^{2}$$
(3)

where  $|H_e(\omega,t)|^2$  is the squared transient transfer function for a single degree-of-freedom oscillator representing the secondary system and can be approximated by:

$$|H_e(\omega,t)| \stackrel{z}{\simeq} \left[ \left( \omega_e^2 - \omega^2 \right)^2 + 4 \varsigma_{et}^2 \omega_e^2 \omega^2 J^{-1} \right]$$
(4)

in which  $S_{et} = S_e [1 - exp(-2S_e \omega_e t)]^{-1}$  and  $S_e$  is the oscillator damping. The transfer function  $|He(\omega,t)|^2$  approaches the steady state value  $|He(\omega)|^2$  as t approaches infinity.

The mean square pseudo-acceleration response,  $\sigma_e^z$ , of the single degree-of-freedom oscillator is then obtained at t=s by the following expression:

$$\sigma_e^2 = \omega_e^4 \int [G_{\frac{1}{2}}(\omega,s) d\omega = \omega_e^4 \int [G_{\frac{1}{2}}(\omega)/H_{\frac{1}{2}}(\omega,s)/f_{\frac{1}{2}}(\omega)] d\omega$$
 (5)

In general, an exact evaluation of Equation 5 will require numerical integration. However, for a smoothly varying spectral density function,  $G_{\chi}(\omega)$ , Equation 5 can be approximated by(6):

$$\sigma_{e}^{2} = \omega_{e} G_{x} (\omega_{e}) \left[ \frac{\pi}{4} - 17 + \int_{0}^{\omega_{e}} G_{x} (\omega) d\omega \right]$$
(6)

where  $S_{es} = S_e [1 - exp(-2S_e \omega_e s)]^{-1}$ .

Equations 6 and 2 provide a suitable means for the computation of the mean square pseudo acceleration response of a single degree-of-freedom oscillator for a specified spectral density input function. Note that in Equation 6 the contribution of the second term is insignificant for single degree-of-freedom oscillators of low frequency but becomes significant for those with higher natural frequencies.

## Primary System Transfer Function H(w)

It is seen from the previous discussion that the estimation of the mean square acceleration response,  $\sigma_e^2$ , requires the evaluation of the acceleration transfer function H(w) of the primary structure. In order to obtain H(w), it is necessary to construct a lumped parameter dynamic model of the primary system. The primary system transfer function H(w) can then either be obtained directly by solving the dynamic equation of motion in the frequency domain (7), or by combining the individual modes from a normal mode analysis. In the frequency domain solution, the transfer function is obtained by solving at each desired frequency a system of linear algebraic equations. If the primary structure is analyzed by the normal mode method, then the acceleration transfer function with respect to a specific location within a primary structure can be expressed as (2):

$$|H(\omega)| = 1 + \sum_{j=1}^{n} 2\omega^{2}(\omega_{j}^{2} - \omega^{2})\phi_{j}f_{j} |H_{j}(\omega)|^{2} + \omega^{4} \sum_{j=1}^{n} \sum_{k=1}^{n} \int_{j}^{n} \phi_{j}f_{k} \phi_{k} H_{j}(\omega) H_{k}^{*}(\omega)$$
(7)

where  $\phi_j$  is the j<sup>#</sup> modal ordinate at the point of interest, n is the total number of degrees of freedom,  $f_j$  is the participation factor in the j<sup>#</sup> mode, and  $\omega_j$  is the j<sup>#</sup> natural frequency. H; ( $\omega$ ) is the complex frequency response function of the j<sup>#</sup> mode and is given by:

$$H_{j}(\omega) = -1(\omega_{j}^{2} \omega^{2} + 2i\varsigma_{j}\omega_{j}\omega)^{-1}$$
(8)

in which  $S_j$  is the damping in the j<sup>#</sup> mode and  $H_{\kappa}^{*}(\omega)$  is the complex conjugate of  $H_{\kappa}(\omega)$ .

Although Equation 7 has the form of a double summation, it has been shown (2) that for a lightly damped system with well separated modal frequencies the main contribution to response comes from the terms for which j=k. Thus Equation 7 can be further simplified to:

$$|H(\omega)|^{2} \simeq 1 + \omega^{2} \sum_{j=1}^{n} \int_{j}^{r} \phi_{j} \left[ 2(\omega_{j}^{2} - \omega^{2}) + \omega^{2} \int_{j}^{r} \phi_{j} J / H_{j}(\omega) \right]^{2}$$
 (9)

Note that in the derivation of the above expressions the stationarity assumption is implied. However, for very light damping, the transient response buildup in a multi-degree-of-freedom system can be approximately accounted for by replacing  $|H_j(\omega)|^2$  in Equation 9 with a time dependent transfer function  $|H_j(\omega,t)|^2$  of the form presented in Equation 4 with  $\omega_e$  replaced by  $\omega_j$  and  $\varsigma_e$  replaced by  $\varsigma_j$ .

## Spectrum-Compatible Power Spectral Density Function

It has been shown that the evaluation of the mean square acceleration response,  $\sigma_e^2$ , is dependent on the input power spectral density (PSD) function  $G_{ii}(\omega)$ . However, in seismic design the earthquake input is often specified not in terms of a PSD function, but rather in terms of a set of smooth design response spectra. Therefore, it is desirable from a practical standpoint to construct a PSD function compatible with the site design spectra.

Based on the principles of random vibration, Vanmarcke and Cornell(8) and Vanmarcke(6) have proposed a workable methodology to construct a spectrum compatible PSD Function. In this method, the maximum relative response,  $y_{max}$ , for a single degree-of-freedom oscillator corresponding to a nonexceedance probability,  $p_q$ , and a strong motion duration, s, is expressed as a multiple of,  $\sigma_s(s)$ , the standard deviation at time, s, of the oscillator response, and a peak factor,  $r_g$ , so that:

$$y_{max} = f_{g} \sigma_{y}(s) \tag{10}$$

The physical interpretation of this relationship is shown in Figure 3. A determination of the peak factor requires the solution of the first passage problem (6, 9, 10), an exact solution of which is not yet available. However, good practical estimates have been proposed by various investigators (6, 11) and it is possible to estimate the peak factor,  $r_g$ , for a high barrier level by:

$$r_{g} = \sqrt{2 \ln [2\gamma_{o} s / \ln(1/\rho_{g})]}$$
 (11)

where  $\Sigma$  is the mean rate of zero crossing of the response given by:

$$Y_0 = \omega_n / 2\pi$$

and  $\omega_n$  is the oscillator natural frequency.

Assuming now that a value from a design response spectrum is equivalent to the maximum response (provided by Equation 10), the following relationship, analogous to Equation 6, can be obtained:

$$(S_a/f_g)^2 = \sigma_y^2(s)\omega_n^4 = G_{ij}(\omega_n)\omega_n\left(\frac{\pi}{4S_t} - I\right) + \int_0^{\omega_n} G_{ij}(\omega) \,d\omega \tag{12}$$

where  $S_i = S[1 - exp(-2S\omega_n S)]^{-1}$  and  $S_a$  is the pseudoacceleration response spectrum ordinate at a frequency  $\omega_n$ and damping value S.

Equation 12 forms the basis of an iterative scheme, as suggested by Vanmarcke(6) where at a frequency  $\omega_n^{i}$  the integral of  $G_{ii}(\omega)$  up to a frequency of  $\omega_n^{i}$  is evaluated numerically and the next ordinate of the PSD function  $G_{ii}(\omega_n^{i+i})$ is then evaluated from Equation 12. Note that for very low frequencies the contribution of the integral in Equation 12 is negligible.

Evaluation of the spectrum compatible PSD function,  $G_{ii}(\omega)$ , is not unique since it depends on a chosen duration, s, of strong motion shaking, the damping value of the prescribed design response spectrum,  $\boldsymbol{\varsigma}$ , and the nonexceedance probability,  $p_a$ .

The duration of strong seismic shaking, s, is a function of various seismic parameters such as earthquake magnitude, source distance, and frequency content. Based on the observation of seismic records, Bolt(12) has provided a practical guide for its determination. Studies have also been performed by Vanmarcke and Lai (13) giving recommendations for the strong motion duration of earthquakes.

Selection of the parameter,  $\varsigma$ , is not unique because seismic input is often specified in terms of a set of response spectra, each curve corresponding to a separate damping value, rather than a single response spectrum. When this is the case, the response spectrum and damping value corresponding to one of the higher damping levels should be used. This is because the estimate of the peak factor,  $r_g$ , as provided by Equation 11, tends to overestimate the peak response for a narrow band process (a characteristic of the response of a lightly damped oscillator); this will yield a less conservative estimate of the resulting PSD input function than when a response spectrum representing lower damping is used.

Selection of a value for the probability of nonexceedance,  $\mathbf{p}_{g}$ , will depend to a great extent on the confidence level one has in the prescribed design response spectra. When based on extensive site studies or on generic response spectra derived from intensive investigation, such as those set forth

by the USNRC in Regulatory Guide 1.60 (14), conservative results may be obtained by treating such spectra as median values and using a nonexceedance probability level of  $p_g = 0.5$ .

#### Kanai-Tajimi Power Spectral Density Function

For wide band excitation processes Kanai and Tajimi (15, 16) have proposed the following form for a PSD function:

$$G_{ii}(\omega) = \left[ 1 + 4 S_g(\omega/\omega_g)^2 \right] G_g \left[ \left( 1 - (\omega/\omega_g)^2 \right)^2 + 4 S_g^2(\omega/\omega_g)^2 \right]$$
(13)

where  $\omega_g$  and  $\zeta_g$  are considered as dominant ground frequency and damping and  $G_{\circ}$  is a measure of intensity of shaking. The values of  $\omega_g = 4\pi$  and  $\zeta_g = .60$  have been recommended for firm soil sites. If the intensity of shaking at a site is known the parameter  $G_{\circ}$  can be evaluated from the following relation:

$$G_o = 4 \varsigma_g \sigma_{ii}^2 / \pi \omega_g (1 + 4 \varsigma_g^2) \tag{14}$$

where  $\sigma_{ii}^2$  is the expected mean square acceleration at the site and can be estimated (6) from the relation:

$$\sigma_{ii} = A / \sqrt{2 \ln (2.8 \Omega s / 2 \pi)}$$
(15)

where A is the expected peak ground acceleration,  $\Omega = 2.1 \omega_g$ and s is the expected duration of strong motion shaking. As will be shown later, in the absence of a prescribed design response spectrum, the Kanai-Tajimi PSD normalized to an expected ground acceleration level can provide a conservative estimate of equipment response for structures founded on firm soil. Equation 15 corresponds to a  $p_g$  value of .5.

## Prediction of Floor Response Spectra

Having determined PSD functions for both the seismic input and building response along with appropriate transfer functions, the building floor response spectra may be determined from random vibration principles from the following:

$$S_e = r_e \, \mathcal{T}_e \tag{16}$$

where  $S_e$  is the maximum acceleration for a single degreeof-freedom oscillator of frequency  $\omega_e$ ,  $\sigma_e$  is the root mean square acceleration response, obtained from Equation 6 and  $r_e$  is the oscillator peak factor which can be evaluated from a relationship similar to that given by Equation 11:

$$r_e = \sqrt{2/n [\omega_e s/\pi ln(1/p_e)]}$$
 (17)

where  $p_{e}$  is the desired nonexceedance probability level of the equipment response. Equation 16 together with Equation 6 provide the basic framework for floor response spectra prediction for a specified probability level  $p_{e}$ .

## PRACTICAL APPLICATION AND RESULTS

In this section, examples of floor response spectra obtained from both the random vibration and time history methods for a soil supported structure are presented. Figure 4 shows the dynamic model of a containment - annulus building for a nuclear power plant where the superstructure is idealized as a lumped mass system and the subgrade is represented by frequency dependent soil springs obtained from a finite element soil structure interaction analysis. An idealized iterated shear profile of the subgrade is shown in Figure 5. The primary system transfer functions are obtained by solving the equations of motion in the frequency domain. Figure 6 shows the absolute value of the acceleration transfer functions at locations A and B of the primary structure. design ground response spectrum is that shown in Figure 7. This spectrum was obtained by normalizing the USNRC Regulatory Guide 1.60 Design Spectrum, for 10 percent damping, to a peak ground acceleration of 0.5g. The duration of strong motion shaking was assumed to be 15 seconds.

For the random vibration approach, the spectrum-compatible PSD function was obtained by iterating on the design response spectrum of Figure 7. The results of this iteration are shown in Figure 8. For comparison purposes, spectrum compatible PSD functions are shown for p equal to both 50 percent and 90 percent nonexceedance values. Also shown is the Kanai-Tajimi PSD function ( $p_{e} = .5$ ) as calculated from Equation 13 and normalized to .5g maximum ground acceleration.

As can be seen from Figure 8, the spectral ordinates of the Kanai-Tajimi PSD function in the lower frequency region (less than 2.0 cps) are somewhat larger than that of response spectrum compatible PSD functions. However, for higher frequency regions (greater than 2.0 cps) the reverse is true. This may be due to the fact that the shape of the NRC response spectrum, from which the compatible PSD functions were derived, is based on actual earthquake records that are representative of both soil and rock sites, whereas the Kanai-Tajimi PSD function is representative of soil sites only. Also, soil sites can exhibit radiational damping values on the order of 70 to 90 percent whereas the Kanai-Taijmi PSD assumes a damping value on the order of 60 percent. However, it should be noted that it is not necessary to have a unique relationship between the two types of PSD functions. The shape functions are presented here only to indicate their respective distribution and magnitude of spectral content.

Having determined the input PSD and the system transfer functions, the 50 percentile and the 90 percentile nonexceedance floor response spectra at locations A and B for equipment damping of 5 percent are computed from Equation 16. These are shown in Figures 9 and 10 for a  $p_g$  of .5 and Figures 11 and 12 for a  $p_g$  of .9.

For the time history approach, an artificial time history, whose response spectrum envelopes the design response spectrum of Figure 7, was used for the analysis. The time history of the floor accelerations were obtained by the Fourier transform technique using the Cooley-Tukey(17) fast Fourier transform (FFT) algorithm. Floor response spectra were then computed from the floor time histories. For purposes of comparison, the results of this method have been shown with the PSD results in Figures 9 through 12.

An inspection of Figures 9 and 10 for  $p_g = .50$  indicates that the 90 percentile floor response spectra obtained from a random vibration approach, in general, exhibits good agreement with the time history results. The time history results indicate peaks and valleys at various frequencies as opposed to the random vibration results which exhibit a smooth curve. This is partly due to the fact that although the time history generated response spectrum envelopes the design spectrum (a requirement of the USNRC for nuclear power plants), the enveloping is not equal at all frequencies and significantly higher energy levels exist at certain frequencies of the time history over that indicated by the design spectrum itself. Also, it should be noted that the response spectrum of two apparently similar time histories which envelope a design spectrum will indicate peaks and valleys at different natural frequencies. The 50 percentile floor response spectra in Figures 9 and 10 represent a lower design load level and hence, a somewhat greater design risk, yet the maximum difference between the 50 percentile and 90 percentile curves is on the order of only 25 percent.

The second value of nonexceedance for the ground input,  $p_g = .9$ , was assumed in this study because of the fact that the USNRC Design Response Spectra is based on response spectra studies of actual earthquake records and is representative of a level equal to a normalized mean value acceleration plus one standard deviation. Assuming a Gaussian distribution, this corresponds to a probability of nonexceedance of 84 percent

Thus the floor response spectra developed by random vibration methods with  $p_g = .9$  approximately reflect the energy level implied in the USNRC Design Response Spectra. Figures 11 and 12 compare this spectra with spectra developed by the time history approach. These results are interesting because they demonstrate the additional safety margin inherent in floor response spectra developed in compliance with the USNRC Regulatory Guide 1.60.

Although not shown here, a study into the effect of the duration of strong motion shaking on the floor response spectra was performed by selecting an alternative duration of s = 10 seconds. The difference in the magnitude of the floor response spectra, between s = 10 seconds and s = 15 seconds was not significant.

A comparison of floor response spectra obtained from a time history analysis and a random vibration analysis using the Kanai-Tajimi PSD function, as shown in Figure 13, indicates that the use of Kanai-Tajimi PSD function provides, for this example, conservative estimates of floor response spectra. Thus in the absence of a specified design response spectra or power spectral density function, a Kanai-Tajimi PSD function normalized to a specified ground acceleration level could provide an adequate estimate of floor response spectra for soil supported structures.

#### CONCLUSIONS

In this paper, a probabilistic method of generating floor response spectra has been reviewed and its application to floor response spectra generation has been demonstrated by means of a seismic analysis of a soil-founded nuclear power plant structure. The results obtained using the probabilistic method stand in good agreement with those obtained using the time history approach. The probabilistic method is simple and requires considerably less computational effort than the time history method. Also, the analysis cost using the probabilistic method is independent of the earthquake record duration, while analyses costs using a time history approach are a direct function of earthquake record duration. Because of cost and time savings available with the probabilistic method, it is possible to make greater use of parametric studies in plant arrangements and to more completely study the effects of variations in soil and structural properties on seismic response. In addition, the method provides a simple technique for determining plant seismic design loads for different levels of seismic risk.

The study was by no means meant to be an all inclusive comparison between time history and probabilistic methods. It does, however, indicate possibilities available both in the design of nuclear power plants, where preliminary studies could establish appropriate values of p, p, and S, thus allowing plant wide use of a probabilistic approach, and in other areas where a simple but effective method for determining seismic response is required to protect public and private facilites.

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Fig. 5 Subgrade Shear Modulus Profile







Fig. 10 Floor Response Spectra  $\bigcirc$  Point B for  $p_g = .5$ 



